J Knowl Econ (2015) 6:905–928
DOI 10.1007/s13132-013-0154-7

National Education and Global Economic Growth: A Synthesis of the Uzawa–Lucas Two-Sector and the Oniki–Uzawa Trade Models

Wei-Bin Zhang

Received: 14 July 2012 / Accepted: 8 February 2013 /
Published online: 22 February 2013

Abstract This paper deals with issues related to global economic growth with capital accumulation and human capital. The economic system is structurally based on the Solow growth model, the Uzawa–Lucas model with education, and the Oniki–Uzawa trade model. We take account of three ways of accumulating human capital: learning by producing, learning by education, and learning by consuming. The model describes a dynamic interdependence among wealth accumulation, human capital accumulation, division of labor, and international trade. The countries differ in preference (such as propensities to save) and to receive education, human capital utilization, and accumulation efficiency and creativity. First, we show that the dynamics of the J-country world economy is described by $2J$ differential equations. Then, we simulate the motion of the global economy with three economies. We also examine the effects of changes in the propensity to receive education, efficiency of learning, and the population upon dynamic paths of the system.

Keywords Growth · International trade · Learning by production · Learning by education · Learning by consuming

Introduction

Recent global economic development is characterized by rapidly increasing mobility of goods and money and wide spread of ideas and education. In order to understand the process of contemporary economic globalization, it is necessary to analyze dynamic interactions of creation and distribution of the two basic production factors, physical wealth and human capital, over time and space. As formation of human capital is closely related to education, it is significant to study the dynamics of education. Easterlin (1981) observed that there were only a few people outside Northwestern Europe and North

W.-B. Zhang (✉)
Ritsumeikan Asia Pacific University, Jumonjibaru, Beppu, Ōita 874-8577, Japan
e-mail: wbz1@apu.ac.jp

America who had any formal education in 1850. Moreover, the spread of formal school seems to have preceded the beginning of modern economic growth. Investment in education has recently become a high priority in almost all developed and developing economies. As the world is globally well-connected both in goods and exchanges in ideas, it is significant to develop a growth model with physical and human capital accumulation in a multicountry framework for understanding mechanisms of modern national as well as global economic growth. The purpose of this study is to build an endogenous multicountry free-trade growth model with endogenous physical and human capital accumulation. As far as a national economy is concerned, we structurally base our model on the Solow growth model and the Uzawa–Lucas growth model with endogenous human capital. We extend the Uzawa–Lucas model to a global world economy with any number of countries with free trade on the basis of the Oniki–Uzawa two-country trade model.

As far as capital mobility and trade are concerned, our model is influenced by the neoclassical growth trade model. As mentioned by Findlay (1984), almost all the trade models developed before the 1960s are static in the sense that the supplies of factors of production are given and do not vary over time; the classical Ricardian theory of comparative advantage and the Heckscher–Ohlin theory are static since labor and capital stocks (or land) are assumed to be given and constant over time. The static trade models with capital movements were originally developed by MacDougall (1960) and Kemp (1961). Since then, economists have made some systematic treatment of capital accumulation or technological changes in the context of international economics. Most of trade models with endogenous capital and/or knowledge are either limited to two-country or small open economies.[1] For instance, Oniki and Uzawa (1965) and Bardhan (1965) examine trade patterns between the two economies in a Heckscher–Ohlin model with fixed saving rates. Deardorff and Hanson (1978) propose a two-country trade mode with different saving rates across countries.[2] As the world consists of many countries, conclusions and insights obtained from examining two-country economies may be invalid for revealing the complexity of trade patterns and economic growth in a multicountry global economy. For instance, if we classify the national economies into developed, newly industrializing, and underdeveloped economies, the two-country modeling framework cannot properly take account of interactions among the three economies. This study extends the traditional two-country framework to any number of national economies.

The neoclassical growth theory considers capital accumulation as the key determinant of per capita economic growth. Nevertheless, it has been argued that human capital is also a key determinant of economic growth.[3] As education is an important way of accumulating human capital, it is important to take account of human capital in explaining trade patterns and economic growth. We develop a model to study how the global integration in capital markets interacts with education. The first formal

[1] See, for instance, Grossman and Helpman (1991), Wong (1995), Jensen and Wong (1998), and Obstfeld and Rogoff (1998).

[2] It should be also mentioned that Brecher et al. (2002), Nishimura and Shimomura (2002), Bold et al. (2003), and Ono and Shibata (2005) study dynamic Heckscher–Ohlin models with technological changes or externalities.

[3] For instance, Hanushek and Kimko (2000), Barro (2001), Krueger and Lindahl (2001), and Castelló-Climent and Hidalgo-Cabrillana (2012).

dynamic growth model with education was proposed by Uzawa (1965). The work by Lucas (1988) has caused a great interest in the issue among economists. Viaenea and Zilcha (2002) propose a two-country model of overlapping generation economies with intergenerational transfers motivated by altruism and investment in human capital. The model predicts that capital market integration enhances government intervention in the provision of public education. Kim and Kim (2000) use a multisector general model where education enhances human capital. They show that international trade, combined with education, can have a positive growth effect by allowing workers to more effectively apply their human capital (without physical capital). The basic concern of the paper is to provide insights into the mobility of workers through school education. The model explains the phenomenon of the outward-oriented Asian NIES which have experienced fast income growth as well as rapid increases in school education and mobility of workers. In another study by Hoffmann (2003), it is shown that, for an initially skilled labor-scarce country, an education subsidy under investment liberalization can jump the economy to a high level equilibrium. It is possible for the economy to sustain its highly developed stage even if the subsidy is removed. The model tries to provide how a poor country can learn from advanced economies through foreign direct investment and education in order to achieve fast growth and guarantee a path of sustainable development. The Uzawa–Lucas model has been extended and generalized in various directions. But most of these works are limited to national economies. Our study extends the Uzawa–Lucas two-sector model into the global economy with any number of countries.

There are many empirical studies about interactions between growth and education. In the last decades, higher education enrollment has increased in many countries.[4] In a classical work on the relation between earnings and education, Mincer (1974) considers individual earnings as a function of years of education, age, experience, and other factors. He finds that, for White males not working on farms, an extra year of education raises the earnings by about 7 %. Nevertheless, earnings appear to be an increasing and decreasing quadratic function of year of work.[5] Psacharopoulos (1994) compares the rates of return to education among 78 countries. There are great differences among countries. In a study on education and income growth, O'Neill (1995) examines the extent to which patterns of human capital convergence can account for observed inequality across countries. The study reveals that, among the developed economies, convergence in education levels reduce income dispersion, while for the world as a whole, incomes diverge despite substantial convergence in education levels. O'Neill argues that this occurs because the rise in the return to education favors the developed countries at the expense of the less developed countries. O'Neill's model is based on two separate modeling frameworks. The first is the Solow model, which predicts that income convergence should be preceded by convergence in physical capital. The convergence happens because of diminishing returns to investment in physical capital. The second is based on the models by, for instance, Romer (1989) and Tamura (1991), which conclude that income convergence results from the flow of technology and human capital from the

[4] See Bergh and Fink (2009).

[5] The estimation neglects the cost of providing the education, the loss of earnings resulting from time for education.

leading economies to the lagging ones. Krueger and Kumar (2004) take account of the differences of education between USA and Europe in explaining the differences in economic development in the 1980s and 1990s. Tselios (2008) study the relationship between income and educational inequalities in the regions of the European Union, using the European Community Household Panel data survey for 94 regions over the period 1995–2000. The research finding suggests a positive relationship between income and educational inequalities. Bergh and Fink (2009) observe two patterns in comparing the provision of higher education and labor market outcomes across countries. The first is that a large share of private providers tends to be associated with higher returns to education. The second pattern is that there does not seem to be a systematic relation between the structure of higher education and the overall degree income inequality. In a recent comparative study of educational attainment across countries by Hendricks (2010), it is found that within-industry variation accounts for at least two thirds of the cross-country and the time series variation in educational attainment.

This study is primarily concerned with the process of physical capital and human capital accumulation and distribution of income and wealth among countries. A national economy is basically described by the Uzawa–Lucas two-sector growth model. As far as capital accumulation and trade pattern determination are concerned, our study follows the Oniki–Uzawa framework.[6] This study deviates from the traditional approach in modeling behavior of households. It is well-known that dynamic optimization models with capital accumulation are associated with analytical difficulties. To avoid these difficulties, this study applies an alternative approach to consumer behavior. The multicountry trade model with capital accumulation and human capital accumulation becomes analytically tractable with the new approach to consumer behavior. The model in this study is a further development of the two models by Zhang. Zhang (1992) proposed a multicountry model with capital accumulation and knowledge creation. The study used the traditional approach to household behavior as in the Solow one-sector growth model, assuming a constant fraction used for saving. The knowledge creation is only through Arrow's learning by doing. This study models the behavior of households in an alternative way and assumes that human capital accumulation is not only through learning by doing, but also through learning by education. Although Zhang (2007) introduced learning by consuming (or creative leisure) into the growth model, the model was limited to a national economy. This study synthesizes the main ideas in the previous two models. In the literature of theoretical economics on inequality, only a few formal models address the issue of wealth inequality across countries. This paper is organized as follows. The "The Multicountry Trade Growth Model with Education" section defines the multicountry model with physical capital and human capital accumulation. The "The World Economic Dynamics" section shows that the world with J economies is described by $2J$ differential equations. The section also simulates the model. The "Comparative Dynamic Analysis" section carries out comparative dynamics analysis with regards to some parameters. The "Conclusions" section concludes the study.

[6] In particular, refer to Ikeda and Ono (1992) for modeling trade patterns.

The Multicountry Trade Growth Model with Education

Following the Uzawa–Lucas model, we consider that each country has one production/industrial sector and one education sector. In describing the production sector, we follow the neoclassical trade framework. It is assumed that the countries produce a homogenous commodity.[7] Most aspects of the production sectors in our model are similar to the neoclassical one-sector growth model.[8] There is only one (durable) good in the global economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of nonproductive inventories held by households. All savings volunteered by households are absorbed by firms. The system consists of multiple countries, indexed by j=1, …, J. Each country has a fixed population, $\bar{N}_j, \ (j = 1, \ldots, J)$. Let prices be measured in terms of the commodity and the price of the commodity be unity. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j-th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t)=r_j(t)$.

We use $K(t)$ to stand for the capital stocks of the world economy. The total capital stock employed by country j, $K_j(t)$, is allocated between the production and education sectors. As the wealth of some countries may be employed by some others, $K_j(t)$ may not equal the wealth owned by country j, $\bar{K}_j(t)$. We use $N_{ji}(t)$ and $K_{ji}(t)$ to stand for the labor force and capital stocks employed by the production sector, respectively, and $N_{je}(t)$ and $K_{je}(t)$ for the labor force and capital stocks employed by the education sector, respectively. As full employment of labor and capital is assumed, we have:

$$K_{ji}(t) + K_{je}(t) = K_j(t), \qquad N_{ji}(t) + N_{je}(t) = N_j(t)$$

in which $N_j(t) \equiv T_j(t)\bar{N}_j$, where $N_j(t)$ is the total work time of the population and $T_j(t)$ is the work time per worker in country j. We rewrite the above relations as follows:

$$n_{ji}(t)k_{ji}(t) + n_{je}(t)k_{je}(t) = k_j(t), \qquad n_{ji}(t) + n_{je}(t) = 1 \tag{1}$$

in which:

$$k_{jq}(t) \equiv \frac{K_{jq}(t)}{N_{jq}(t)}, \quad n_{jq}(t) \equiv \frac{N_{jq}(t)}{N_j(t)}, \quad k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad q = i, e.$$

[7] This follows the Oniki–Uzawa trade model and its various extensions with one capital goods (Ikeda and Ono 1992).

[8] See, for instance, Burmeister and Dobell (1970), Azariadis (1993), and Barro and Sala-i-Martin (1995).

Production Sectors

Let $H_j(t)$ stand for the level of human capital of the population in country j. We use $H_j^{m_j}(t)N_{ji}(t)$ to measure the qualified labor force, where m_j is a positive parameter measuring how effectively country j applies human capital in the production sector. We assume that production is to combine the qualified labor and physical capital. We use the conventional production function to describe a relationship between inputs and output. We specify the production functions as follows

$$F_{ji}(t) = A_{ji}K_{ji}^{\alpha_{ji}}(t)\left(H_j^{m_j}(t)N_{ji}(t)\right)^{\beta_{ji}}, \qquad A_{ji}, \alpha_{ji}, \beta_{ji} > 0, \alpha_{ji} + \beta_{ji} = 1$$

where A_{ji}, α_{ji}, and β_{ji} are positive parameters. The production functions are neoclassical and homogeneous of degree 1 with the inputs. In this study, we assume human capital endogenously changeable and the total factor productivities A_{ji} fixed. It is well-known that, among economists, there have been conflicting views on the importance of differences in human capital and total factor productivity in accounting for income differences across countries. For instance, Erosa et al. (2010) built a model of endogenous human capital accumulation with education to explain the variation in per capita income across countries. Heterogeneous households make investments in schooling quantity and quality. The study qualifies the importance of differences in human capital and total factor productivity. It shows that human capital accumulation strongly amplifies total factor productivity differences across countries. This implies that the total factor productivities should be endogenous.

The rate of interest and wage rate are determined by markets. Hence, for any individual firm, $r(t)$ and $w_j(t)$ are given at each point of time. The production sector chooses the two variables $K_{ji}(t)$ and $N_{ji}(t)$ to maximize its profit. The marginal conditions are given by:

$$r(t) + \delta_{jk} = \frac{\alpha_{ji}F_{ji}(t)}{K_{ji}(t)} = \alpha_{ji}A_{ji}H_j^{m_j\beta_{ji}}k_{ji}^{-\beta_{ji}}, \qquad w_j(t)\,\frac{\beta_{ji}F_{ji}(t)}{N_{ji}(t)} = \beta_{ji}A_{ji}H_j^{m_j\beta_{ji}}k_{ji}^{\alpha_{ji}} \tag{2}$$

where δ_{jk} is depreciation rate of physical capital.

Education Sector

We assume that the education sector is also characterized by perfect competition. Students are supposed to pay the education fee $p_j(t)$ per unit of time. The education sector pays teachers and capital with the market rates. The cost of the education sector is given by $w_j(t)N_{je}(t) + r(t)K_{je}(t)$. The total education service is measured by the total education time received by the population, $T_{je}N_0$. The production function of the education sector is assumed to be a function of $K_{je}(t)$ and $N_{je}(t)$. We specify the production function of the education sector as follows:

$$F_{je}(t) = A_{je}K_{je}^{\alpha_{je}}\left(H^{m_j}N_{je}\right)^{\beta_{ej}}, \qquad \alpha_{ej}, \beta_{je} > 0, \alpha_{je} + \beta_{je} = 1 \tag{3}$$

where A_{je}, α_{je}, and β_{je} are positive parameters. For the given $p_j(t)$, $H_j(t)$, $r(t)$, and $w_j(t)$, the marginal conditions for the education sector are given by:

$$r+\delta_{jk}=\frac{\alpha_{je}p_jF_{je}}{K_{je}}=\alpha_{je}A_{je}p_jH_j^{m_j\beta_{je}}k_e^{-\beta_e},\quad w_j=\frac{\beta_{je}p_jF_{je}}{N_{je}}=\beta_{je}A_{je}p_jH_j^{m_j\beta_{je}}k_{je}^{\alpha_{je}}. \tag{4}$$

The demand for labor force for the given price of education, wage rate, and level of human capital is

$$N_{je}=K_{je}\left(\frac{\beta_{je}A_{je}p_jH_j^{m_j\beta_{je}}}{w_j}\right)^{1/\alpha_{je}}.$$

We see that the demand for labor force from the education sector increases in the price and levels of human capital and physical capital and decreases in the wage rate.

Consumer Behaviors and Wealth Dynamics

Consumers make decisions on choice of consumption levels of services and commodities as well as on how much to save. Let $\bar{k}_j(t)$ stand for the per capita wealth in country j. We have $\bar{k}_j(t)=\bar{K}_j(t)/\bar{N}_j$. Per capita current income from the interest payment $r(t)\,\bar{k}_j(t)$ and the wage payment $T_j(t)w_j(t)$ is given by $y_j(t)=r(t)\bar{k}_j(t)+T_j(t)w_j(t)$.

We call $y_j(t)$ the current income in the sense that it comes from consumers' payment for human capital and efforts and consumers' current earnings from ownership of wealth. The sum of money that consumers are using for consuming, saving, and education are not necessarily equal to the current income because consumers can sell wealth to pay, for instance, the current consumption if the temporary income is not sufficient for buying food and touring the country. People may live not only on the interest payment but also have to spend some of their wealth. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\hat{y}_j(t)=y_j(t)+\bar{k}_j(t)=(1+r(t))\bar{k}_j(t)+T_j(t)w_j(t). \tag{5}$$

It should be noted that the value $\bar{k}_j(t)$ (i.e., $p(t)\bar{k}_j(t)$ with $p(t)=1$), in Eq. 5 is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider $\bar{k}_j(t)$ the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t, the consumer has the total amount of disposable income equaling $\hat{y}_j(t)$. The consumer may use the disposable income for different purposes, such as buying food, travelling, saving, or investing in education. In the growth literature, for instance, in the Solow model, the saving is out of the current income, $y_i(t)$, while in this study, the saving is out of the disposable income, $\hat{y}_j(t)$. In reality, when one makes a decision, the decision is not only dependent on how much one earns from the wage and interest payment, but also dependent on how much wealth one holds. For instance, according to the Solow model, an extremely rich individual should consume almost nothing when he is not working and rate of interest is almost zero.

The consumer distributes the disposable income among saving, $s_j(t)$; consumption of goods, $c_j(t)$; and education, $p_j(t)T_{je}(t)$. The budget constraint is:

$$c_j(t) + s_j(t) + p_j(t)T_{je}(t) = \widehat{y}_j(t) = (1 + r(t))\bar{k}_j(t) = T_j(t)w_j(t). \tag{6}$$

Let $T_{je}(t)$ stand for the time spent on education. We assume that the total available time is distributed between work and education. The consumer is faced with the following time constraint:

$$T_j(t) = T_{je}(t) = T_0$$

where T_0 is the total available time for work and study. Substituting this function into the budget constraint 4 yields:

$$c_j(t) + s_j(t) + \left(p_j(t) + w_j(t)\right)T_{je}(t) = \bar{y}_j(t) \equiv (1 + r(t))\bar{k}_j(t) + T_0 w_j(t). \tag{7}$$

In our model, at each point of time, consumers have three variables, the level of consumption, the level of saving, and the education time, to decide. We assume that consumers' utility function is a function of level of goods, $c_j(t)$, level of saving, $s_j(t)$, and education service, $T_{je}(t)$, as follows:

$$U_j(t) = c_j^{\xi_{j0}} s_j^{\lambda_{j0}} T_{je}^{\eta_{j0}}(t), \quad \xi_{j0}, \lambda_{j0}, \eta_{j0} > 0 \tag{8}$$

where ξ_{j0} is called the propensity to consume, λ_{j0} the propensity to own wealth, and η_{j0} the propensity to receive education. This utility function is initially proposed by Zhang about two decades ago. A detailed explanation of the approach and its applications to different problems of economic dynamics are provided in Zhang (2007). Maximizing $U_j(t)$ subject to Eq. 7 yields:

$$c_j(t) = \xi_j \bar{y}_j(t), \quad s_j(t) = \lambda_j \bar{y}_j(t), \quad \left(p_j(t) + w_j(t)\right)T_{je}(t) = \eta_j \bar{y}_j(t) \tag{9}$$

where $p_j(t)+w_j(t)$ is the opportunity cost in country j and:

$$\xi_{j0} \equiv \rho_j \xi_{j0}, \quad \lambda_{j0} \equiv \rho_j \lambda_{j0}, \quad \eta_{j0} \equiv \rho_j \eta_{j0}, \quad \rho_j \equiv \frac{1}{\xi_{j0} + \lambda_{j0} + \eta_{j0}}.$$

The demand for education is given by $T_{je} = \eta_j \bar{y}_j / \left(p_j + w_j\right)$. The demand for education decreases in the opportunity cost and the wage rate and increases in $\bar{y}_j$. An increase in the propensity to receive education increases the education time when the other conditions are fixed. We see that it is not only price of education or wage but the sum of the price and wage (that is, the opportunity cost) that matters when the household considers the cost of education. As any factor is related to all the other factors over time, it is difficult to see how one factor affects any other variable over time in the global economy. We will demonstrate the complicated interactions over time by simulation.

According to the definitions of $s_j(t)$, the wealth accumulation of the representative household in country j is given by:

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{10}$$

This equation simply states that the change in wealth is equal to the saving minus the dissaving.

Human Capital Accumulation Equation

In a recent article on cognitive skills and economic development by Hanushek and Woessmann (2008), it is found that the cognitive skills of the population, rather than mere school attainment, are strongly related to economic growth, individual earnings, and distribution. Their empirical evidence shows the importance of different levels of skill and the robustness of the relationship between skills and growth. In particular, their international comparisons incorporating data on cognitive skills demonstrate much larger deficits in developing countries than commonly obtained just from school enrollment and attainment. These conclusions imply that it is necessary to generalize the traditional models of human capital formation which often assume a single source of human capital accumulation. In this study, we synthesize different sources of learning in an integrated framework. We take account of three sources of improving human capital, through education, "learning by producing" and "learning by leisure." Arrow (1962) first introduced learning by doing into the growth theory[9]; Uzawa (1965) took account of trade-offs between investment in education and capital accumulation, and Zhang (2007) introduced the impact of consumption on human capital accumulation (via the so-called creative leisure) into the growth theory. We propose the following human capital accumulation equation:

$$\dot{H}_j = \frac{\upsilon_{je} F_{je}^{a_{je}} \left(H_j^{m_j} T_{je} \overline{N}_j \right)^{b_{je}}}{H_j^{\pi_{je}} \overline{N}_j} + \frac{\upsilon_{ji} F_{ji}^{a_{ji}}}{H_j^{\pi_{ji}} \overline{N}_j} + \frac{\upsilon_{jh} C_j^{a_{jh}}}{H_j^{\pi_h} \overline{N}_j} - \delta_{jh} H_j \tag{11}$$

where $\delta_{jh}(>0)$ is the depreciation rate of human capital, $\upsilon_{je}, \upsilon_{ji}, \upsilon_{jh}, \alpha_{je}, b_{je}, \alpha_{ji}$, and α_{jh} are the non-negative parameters. The signs of the parameters π_{je}, π_{ji}, and π_{jh} are not specified as they may be either negative or positive. The equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation. The term:

$$\frac{\upsilon_{je} F_{je}^{a_{je}} \left(H_j^{m_j} T_{je} \overline{N}_j \right)^{b_{je}}}{H_j^{\pi_{je}} \overline{N}_j}$$

describes the contribution to human capital improvement through education. Human capital tends to increase with an increase in the level of education service, F_{je}, and in the (qualified) total study time, $H_j^{m_j} T_{je} \overline{N}_j$. The population, $\overline{N}_j$, in the denominator measures the contribution in terms of per capita. The term $H^{\pi_{je}}$ indicates that, as the

[9] The part of learning by producing is based on Zhang (1993).

level of human capital of the population increases, it may be more difficult (in the case of π_{je} being large) or easier (in the case of π_{je} being small) to accumulate more human capital via formal education. We take account of learning by producing effects in human capital accumulation by the term $\upsilon_{ji}F_{ji}^{a_{ji}}\Big/H_j^{\pi_{ji}}$. This term implies that contribution of the production sector to human capital improvement is positively related to its production scale F_{ji} and is dependent on the level of human capital. The term $H^{\pi_{ji}}$ takes account of returns to scale effects in human capital accumulation. The case of $\pi_{ji}>(<)0$ implies that, as human capital is increased, it is more difficult (easier) to further improve the level of human capital. We take account of learning by consuming by the term $\upsilon_{jh}C_j^{a_{jh}}\Big/H_j^{\pi_{jh}}\overline{N}_j$. This term can be interpreted similarly as the term for learning by producing.

In the literature on education and economic growth, it is assumed that human capital evolves according to the following equation (Lucas 1988):

$$\dot{H}(t) = H^{\eta}(t)G(T_e(t))$$

where the function G is increasing as the effort rises with $G(0)=0$. In the case of $\eta<1$, there is diminishing return to the human capital accumulation. It seems reasonable to consider diminishing returns in human capital accumulation: people accumulate it rapidly early in life, then less rapidly, then not at all—as though each additional percentage increment were harder to gain than the preceding one. Solow adapts the Uzawa formation to the following form:

$$\dot{H}(t) = H(t)\kappa T_e(t).$$

This is a special case of the preceding equation. The new formation implies that, if no effort is devoted to human capital accumulation, then $\dot{H}(0) = 0$ (human capital does not vary as time passes); if all effort is devoted to human capital accumulation, then $g_H(t)=\kappa$ (human capital grows at its maximum rate; this results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$. To achieve a given percentage increase in $H(t)$ requires the same effort. As remarked by Solow (2000), the previously shown formulation is very far from a plausible relationship. If we consider the equation as a production for new human capital (i.e., $\dot{H}(t)$) and if the inputs are already accumulated human capital and study time, then this production function is homogenous of degree 2. It has strong increasing returns to scale and constant returns to $H(t)$ itself. It can be seen that our approach is more general to the traditional formation with regard to education. Moreover, we treat teaching also as a significant factor in human capital accumulation. Efforts in teaching are neglected in the Uzawa–Lucas model. We should mention another approach by Krueger and Kumar (2004). Households may obtain either a concept-based general education or skill-specific vocational education. They assume that it is costly for households to obtain general education which enables them to operate production technologies. It is shown that an economy which emphasizes vocational education will grow slower in equilibrium than the one that favors general education. Through their analytical conclusions, they suggest that the European education policies which favored vocational education might have worked well during the 1960s and 1970s when the available technologies change slowly. Nevertheless, in the 1980s and 1990s, when new technologies were innovated rapidly, the USA had experienced more rapid economic growth. In a general sense, our equation of human capital accumulation takes account of education and vocational education (i.e., learning by producing) in the Krueger–Kumar approach.

Balance of Demand and Supply

For the education sector, the demand and supply balances at any point of time:

$$T_{je}\overline{N}_j = F_{je}(t). \tag{12}$$

The total capital stocks employed by the world are equal to the wealth owned by the world. That is:

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \overline{k}_j(t)\overline{N}_j. \tag{13}$$

The world production is equal to the world consumption and world net savings. That is:

$$C(t) + S(t) - K(t) + \sum_{j=1}^{J} \delta_{kj} K_j(t) = F(t) \tag{14}$$

where:

$$C(t) \equiv \sum_{j=1}^{J} c_j(t)\overline{N}_j, \quad S(t) \equiv \sum_{j=1}^{J} s_j(t)\overline{N}_j, \quad F(t) \equiv \sum_{j=1}^{J} F_j(t).$$

The trade balances of the economies are given by:

$$E_j(t) = \left(\overline{K}_j(t) - K_j(t)\right) r(t). \tag{15}$$

When $E_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $E_j(t)$ is zero, country j's trade is in balance.

We have thus built the model with trade, economic growth, and physical and human capital accumulation in the world economy in which the domestic markets of each country are perfectly competitive, international product and capital markets are freely mobile, and labor is internationally immobile.

The World Economic Dynamics

We first show that, in a general case, the dynamics of the world economy can be expressed by a $2J$ dimensional differential equations system.

Lemma 1

The dynamics of the world economy is governed by the following 2*J* dimensional differential equations system with $k_{1i}(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$ and $(H_j(t)) \equiv (H_1(t), \cdots, H_J(t))$, as the variables:

$$\begin{aligned}
\dot{k}_{1i}(t) &= \bar{\Lambda}_1\left(k_{1i}(t), (H_j(t)), \{\bar{k}_j(t)\}\right), \\
\dot{\bar{k}}_j(t) &= \bar{\Lambda}_j\left(k_{1i}(t), (H_j(t)), \{\bar{k}_j(t)\}\right), \quad j = 2, \ldots, J, \\
\dot{H}_j(t) &= \Lambda_j\left(k_{1i}(t), (H_j(t)), \{\bar{k}_j(t)\}\right), \quad j = 1, \ldots, J,
\end{aligned}$$

in which $\bar{\Lambda}_j$ and Λ_j are unique functions of $k_{1i}(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, defined in the Appendix. For any given positive values of $k_{1i}(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$ at any point of time, the other variables are uniquely determined by the following procedure: $k_{ji}(t)$ by Eq. 18→$k_{je}(t)$ by Eq. 20→$r(t)$ and w_i by Eq. 19→$p(t)$ by Eq. 21→ $\bar{k}_1(t)$ by Eq. 29→$K_j(t)$ by Eq. 29→$k_j(t)$ by Eq. 26→$T_j(t)$ by Eq. 27→$T_{je}(t)$ by Eq. 25→ $N_j(t) = T_j(t)\bar{N}_j(t) \to n_{ji}(t)$ and $n_{je}(t)$ by Eq. 22→$N_{ji}=n_{ji}N_j$ and $N_{je}(t) = n_{je}(t)N_j(t) \to K_{ji}(t) = k_{ji}(t)N_{ji}(t)$ and $K_{je}(t) = k_{je}(t)N_{je}(t) \to F_{ji}(t)$ by Eq. 2→ $F_{je}(t)$ by Eq. 12→$\bar{y}_j(t)$ by Eq. 7→$c_j(t)$ and $s_j(t)$ by Eq. 9.

We have the dynamic equations for the world economy with any number of countries. The system is nonlinear and is of high dimension. It is difficult to generally analyze the behavior of the system. We simulate the model to illustrate the properties of the dynamic system. We specify the parameters as follows:

$$\begin{aligned}
&\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 100 \\ 400 \\ 1,000 \end{pmatrix}, \begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \end{pmatrix} = \begin{pmatrix} 0.9 \\ 0.8 \\ 0.7 \end{pmatrix}, \begin{pmatrix} A_{1e} \\ A_{2e} \\ A_{3e} \end{pmatrix} = \begin{pmatrix} 0.9 \\ 0.8 \\ 0.7 \end{pmatrix}, \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.5 \\ 0.4 \end{pmatrix}, \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.12 \\ 0.18 \\ 0.2 \end{pmatrix}, \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.75 \\ 0.7 \end{pmatrix} \\
&\begin{pmatrix} \eta_{10} \\ \eta_{20} \\ \eta_{30} \end{pmatrix} = \begin{pmatrix} 0.008 \\ 0.007 \\ 0.006 \end{pmatrix}, \begin{pmatrix} v_{1e} \\ v_{2e} \\ v_{3e} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.7 \\ 0.5 \end{pmatrix}, \begin{pmatrix} v_{1i} \\ v_{2i} \\ v_{3i} \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2 \\ 1.7 \end{pmatrix}, \begin{pmatrix} v_{1h} \\ v_{2h} \\ v_{3h} \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.6 \\ 0.5 \end{pmatrix}, \begin{pmatrix} a_{1e} \\ a_{2e} \\ a_{3e} \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.4 \\ 0.55 \end{pmatrix}, \begin{pmatrix} b_{1e} \\ b_{2e} \\ b_{3e} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.55 \\ 0.6 \end{pmatrix} \\
&\begin{pmatrix} a_{1i} \\ a_{2i} \\ a_{3i} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.45 \\ 0.5 \end{pmatrix}, \begin{pmatrix} a_{1h} \\ a_{2h} \\ a_{3h} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.15 \\ 0.2 \end{pmatrix}, \begin{pmatrix} b_{1h} \\ b_{2h} \\ b_{3h} \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.35 \\ 0.4 \end{pmatrix}, \begin{pmatrix} \pi_{1e} \\ \pi_{2e} \\ \pi_{3e} \end{pmatrix} = \begin{pmatrix} -0.2 \\ -0.15 \\ -0.1 \end{pmatrix}, \begin{pmatrix} \pi_{1i} \\ \pi_{2i} \\ \pi_{3i} \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.75 \\ 0.8 \end{pmatrix} \\
&\begin{pmatrix} \pi_{1h} \\ \pi_{2h} \\ \pi_{3h} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.15 \\ 0.2 \end{pmatrix}, \alpha_{ji} = 0.3, \quad \alpha_{je} = 0.34, \quad T_0 = 1, \quad \delta_k = 0.05, \quad \delta_{ih} = 0.04, \quad \delta_{2h} = 0.05, \quad \delta_{3h} = 0.06
\end{aligned} \tag{16}$$

Country 1, 2, and 3's populations are, respectively, 100, 400, and 1,000. Country 3 has the largest population. Country 1, 2, and 3's total productivities of the industrial sectors, A_{je}, are, respectively, 0.9, 0.8, and 0.7. Country 1, 2, and 3's total productivities of the education sectors, $A_{je,}$, are, respectively, 0.9, 0.8, and 0.7. Country 1, 2, and 3's efficiency parameter of human capital, m_j, are, respectively, 0.6, 0.4 and 0.4. Country 1 applies human capital mostly effectively; country 2 next and country 3 least effectively. We call the three countries, respectively, as developed, industrializing, and underdeveloped economies (DE, IE, and UE). We specify the values of the parameters, α_{ji}, in the Cobb–Douglas productions approximately equal to 0.3.[10] The DE's learning by doing parameter, π_{i1}, is the highest among the countries. The returns to scale parameters in learning by doing, π_j, are all positive, which implies that human capital accumulation exhibits decreasing returns to scale in learning by doing. The depreciation rates of physical capital and knowledge are specified to about 0.05. The DE's propensity to save is 0.8 and the UE's propensity to save is 0.7. The value of the IE's propensity is between the two other countries. The DE's propensity to obtain education is 0.008 and the UE's propensity to obtain education is 0.007. The value of the IE's propensity is between the two other countries. Under Eq. 16, the dynamic system has a unique equilibrium as follows:

$$\begin{array}{l} r = 0.036, \quad E_1 = 30.53, \quad E_2 = -2.90, \quad E_3 = -27.63, \quad F_{1i} = 326.96, \quad F_{2i} = 621.52 \\ F_{3i} = 1,016.42, \quad F_{1e} = 7.31, \quad F_{2e} = 12.18, \quad F_{3e} = 18.71, \quad H_1 = 4.40, \quad H_2 = 1.71, \quad H_3 = 1.06 \\ K_1 = 1,172.8 \quad K_2 = 2,210.15, \quad K_3 = 3,610.45, \quad N_1 = 93.92, \quad N_2 = 389.86, \quad N_3 = 984.44, \\ N_{1i} = 92.12, \quad N_{2i} = 383.21, \quad N_{3i} = 968.52, \quad N_{1e} = 1.80, \quad N_{2e} = 6.66, \quad N_{3e} = 15.92, \\ K_{1i} = 1,145.88, \quad K_{2i} = 2,164.96, \quad K_{3i} = 3,540.52, \quad K_{1e} = 26.92, \quad K_{2e} = 45.19, \quad K_{3e} = 69.93, \\ k_{1i} = 12.44, \quad k_{2i} = 5.65, \quad k_{3i} = 3.66, \quad k_{1e} = 14.95, \quad k_{2e} = 6.79, \quad k_{3e} = 4.39, \quad f_{1i} = 3.57, \\ f_{2i} = 1.62, \quad f_{3i} = 1.05, \quad f_{1e} = 4.06, \quad f_{2e} = 1.83, \quad f_{3e} = 1.17, \quad p_1 = 0.82, \quad p_2 = 0.83, \\ p_3 = 0.83, \quad w_1 = 2.50, \quad w_2 = 1.14, \quad w_3 = 0.73, \quad \bar{k}_1 = 20.18, \quad \bar{k}_2 = 5.33, \quad \bar{k}_3 = 2.84, \\ T_{1e} = 0.060, \quad T_{2e} = 0.025, \quad T_{3e} = 0.015, \quad c_1 = 3.03, \quad c_2 = 1.27, \quad c_3 = 0.81. \end{array} \tag{17}$$

The interest rate is about 3.6 %. The DE is in trade surplus and the other two economies in trade deficit. The output levels of the DE, IE, and UE's industrial sectors are, respectively, 326.96, 621.52, and 1,016.42. The DE, IE, and UE's levels of human capital are, respectively, 4.40, 1.71, and 1.06. The price of the education in the DE is lowest, even though the levels of human capital, wealth, and consumption per capita are highest among the three economies. The worker in the DE works less hours but studies more than the workers in the other two countries.

It is straightforward to calculate the six eigenvalues at equilibrium as follows:

$$-0.20, \quad -0.18, \quad -0.12, \quad -0.08, \quad -0.05, \quad -0.04.$$

We see that the equilibrium is locally stable. This implies that, if we start with different initial states not far away from the equilibrium point, the system approaches to the equilibrium point in the long term. In Fig. 2, we plot the motion of the system with the following initial conditions not very far from the equilibrium:

$$k_{1i}(0) = 11, \quad \bar{k}_2(0) = 4.5, \quad \bar{k}_3(0) = 2, \quad H_1(0) = 3.5, \quad H_2(0) = 1.9, \quad H_3(0) = 0.5.$$

The system approaches to its equilibrium in the long term. Before the system approaches its equilibrium, in the DE and UE, human capital is increased over time, but in the IE, human capital is reduced. In the DE, education time falls, but in the IE and UE, the education time rises. In the DE, the levels of per capita consumption and wealth fall, but in the IE and UE, the levels of per capita consumption and wealth rise. It should be noted that, in the long term, the differences between the DE and the UD are not much reduced. This implies occurrence of divergence if the UE does not improve the leaning ability and other aspects such as the propensity to save.

[10] The value is often used in empirical studies. For instance, Abel and Bernanke (1998).

It should be noted that much of the discussion of income convergence in the literature of economic growth and development is based on the insights from analyzing models of closed economies (Barro and Sala-i-Martin 1995). The main reason for this is that there are few growth models with endogenous wealth and trade on the basis of microeconomic foundation. For instance, the Solow model of closed economies predicts that convergence in income levels among closed countries is achieved by faster accumulation of physical capital in the poor countries. Nevertheless, if poor countries are opened to trade, the convergence may be stopped. As shown in Fig. 1, different countries will not experience convergence in per capita income, consumption, and wealth in the long term as they are different in preferences and total productivities. By the way, it should be note that Tamura (1991, pp. 522–523) holds: "Income convergence arises from human capital convergence … Individuals with below-average human capital agents gain disproportionately by the external effect compared with above-average human capital agents. … Convergence arises because below-average human capital agents gain the most from learning." In Tamura's approach, there is no depreciation of human capital. Hence, a below-average human capital agent will always catch up in the long term as the above-average human capital agents will slow down human capital accumulation. In our model, human capital does not converge in the long term because there is also depreciation of human capital (which is assumed to be proportional to the current level of human capital). Hence, if a country has no ability to learn fast, it can never catch up.

Comparative Dynamic Analysis

We simulated the motion of the dynamic system. It is important to ask questions such as how changes in one country's conditions will affect the global economy and different countries.

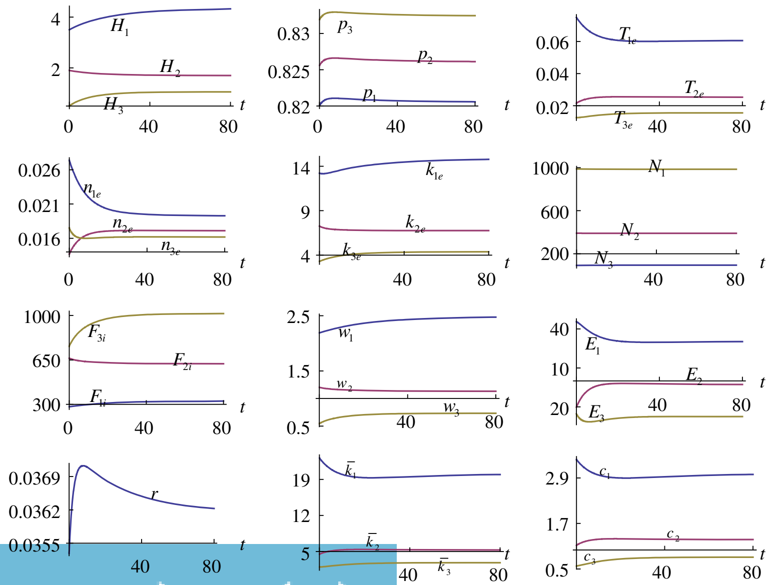

Fig. 1 The motion of some variables

A Rise in the Developed Economy's Propensity to Receive Education

First, we examine the case that all the parameters, except the DE's propensity to receive education, η_{10}, are the same as in Eq. 16. We change the propensity to obtain education as follows: η_{10}=0.008⇒0.01. The simulation results are demonstrated in Fig. 2. In the plots, a variable $\overline{\Delta} x_j(t)$ stands for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value. We will use the symbol $\overline{\Delta}$ with the same meaning when we analyze other parameters. As the DE's propensity to obtain education is increased, the households of the DE raise their education time more than 20 % and the level of human capital is increased. As the demand for education is increased and the education fee is slightly affected, the labor share of the education sector is increased over time. As the work time is reduced and the level of human capital is increased but not much initially, the output of the industrial sector falls initially. But as human capital is improved, the output is increased in the long term. The trade condition of the DE is worsened initially but improved in the long term; the trade conditions of the IE and UE are improved initially but worsened in the long term. Hence, for the DE, the short run effects of a rise in the propensity to obtain education are not desirable in terms of consumption, wealth, and the output level of the production sector, but the long term effects are positive. As the rate of interest rises initially, the output levels of the industrial sectors in the IE and UE are reduced, but in the long term, the variables are slightly affected. It should be noted that different studies provide quite different conclusions about relations between growth and education. For instance, Benhabib and Spiegel (1994) show that changes in educational attainment have no connection on growth. Barro (1991) and Mankiw et al. (1992) propose a positive relationship between growth and educational attainment. Although our special case shows a positive relation between the two variables for the DE in the long term, our result is obtained under the assumption that education makes a significant contribution to growth of human capital and more human capital implies higher productivity. If the term of contribution to human capital in the human capital accumulation is omitted (in the case that education is only for signaling[11]) or efficiency of human capital is extremely low (in the case that a society greatly misallocates skills[12]), then the relation between the two variables in our model may not be positive in the long term. It should be noted that, as explained by Chanda (2008), over the last three decades, returns to higher education have increased while the household savings rate has fallen to almost zero in the USA. Chanda builds a representative agent model where savings fall as an outcome of an exogenously driven increase in the return to education. The model shows that a part of the decline in savings may reflect a relative reallocation of the resources from wealth accumulation to human capital accumulation. Our analysis result hints on other possible factors for explaining the falling saving rate as if return from education is not low, a rise in the education investment will not reduce wealth as well consumption level in the long term.

[11] See, for instance, Spence (1973), Arrow (1973), and Stiglitz (1975). Another issue about education and economic efficiency is the so-called overeducated (for instance, Jauhiainen 2011).

[12] Murphy et al. (1991) and Gelb et al. (1991) present empirical evidence about misallocation of skills. Baumol (1990) discusses how an economy may stagnate because rent-seeking rewards talent more than entrepreneurship and the most talented people would move away from productive sectors.

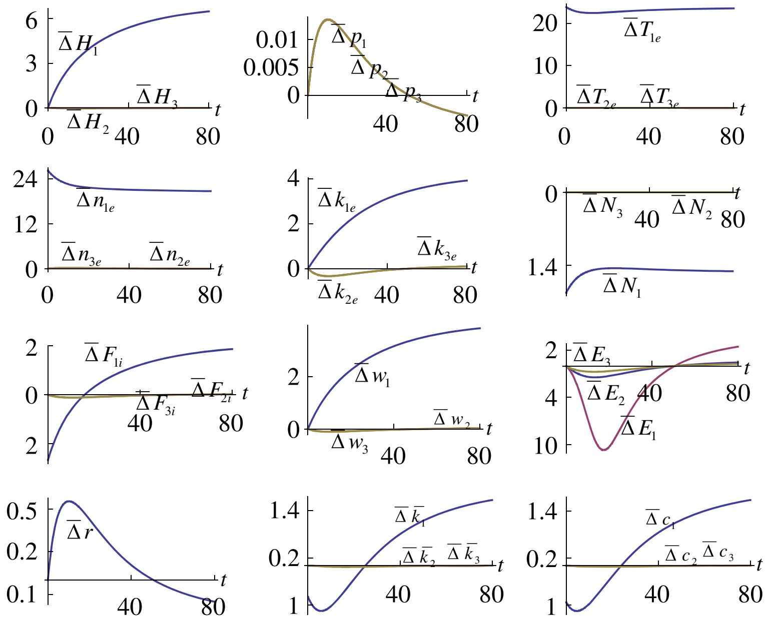

Fig. 2 A rise in the DE's propensity to receive education

A Rise in the Developed Economy's Population

It has been observed that the effect of population growth varies with the level of economic development and can be positive for some developed economies. Theoretical models with human capital predict situation-dependent interactions between population and economic growth (see, Ehrlich and Lui 1997; Galor and Weil 1999; Boucekkine et al. 2002). We now examine the effects of population growth on the world and national economies. We increase the DE's population as follows: $\overline{N}_1 = 100 \Rightarrow 150$. The simulation results are demonstrated in Fig. 3. As the DE increases the population, the country's human capital falls. It should be noted that, in some studies, a rise in the population will result in increases in human capital. In fact, our model also includes this possibility. In our simulation, all the three sources of learning exhibit decreasing returns to scale. If some learning source like education in the DE exhibits increasing returns to scale, then a rise in the DE's population will increase the DE's level of human capital. The levels of human capital in the IE and UE are slightly increased partly because of the fall in the education fees and rises in the income. The DE's output of the industrial sector is increased. The increase in the supply of capital goods reduces the rate of interest.

A Rise in the Underdeveloped Economy's Population

We increase the UE's population as follows: $\overline{N}_3 = 1,000 \Rightarrow 1,200$. The effects are plotted in Fig. 4. It is important to compare differences in the effects of change in population between the DE and UD. A rise in the population in the DE reduces the rate of interest and the fees of education and education times in all the countries; a rise

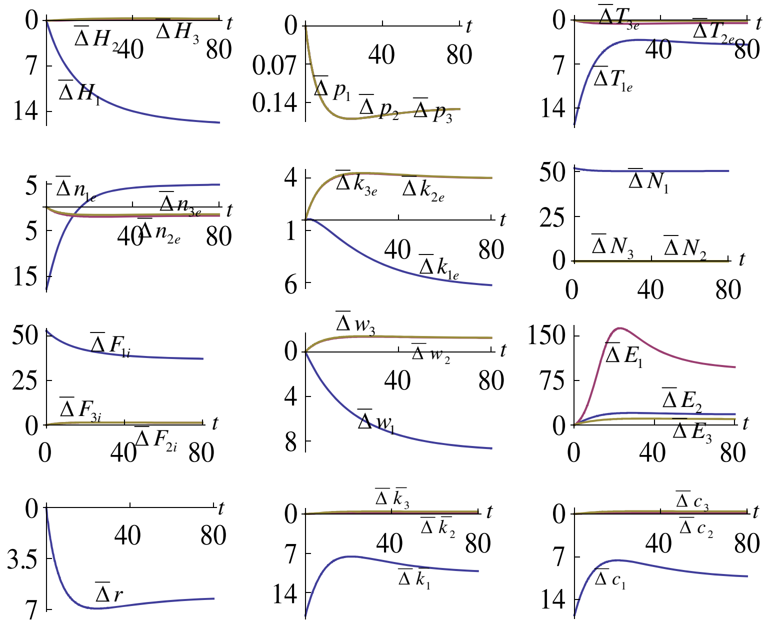

Fig. 3 A rise in the DE's population

in the population in the UE increases the rate of interest and the fees of education in all the three countries and reduces the UE's education time, but increases the IE's and DE's education times. A rise in the DE's population raises the levels of the industrial sectors in the three economies and raises the IE and the UE's wage rates, but a rise in the UE's population raises the UE's output level, reduces the DE's and IE's output levels, and reduces the age rates in the three economies. In the case of the DE's population increase, the DE's per capita wealth and consumption levels are reduced, but the UE's per capita wealth and consumption levels are increased. In the case of the UE's population increase, the UE's and IE's per capita wealth and consumption levels are reduced, but the DE's per capita wealth and consumption levels are increased. In the long term, a rise in the DE's population tends to benefit the other countries, but a rise in the UE's population tends to worsen some other countries' living conditions.

A Rise in the Underdeveloped Economy's Propensity to Save

We now examine the effects of a rise in the UE's propensity to save as follows: λ_{30}=0.7⇒0.75. The effects are plotted in Fig. 5. As the UE increases the propensity to save, the rate of interest and the education fees in the three economies are reduced. The fall of rate of interest is associated with the increases of the wage rates and output levels in the three economies. The education time of the UE falls initially as more income is devoted to wealth accumulation, but the education time rises in the long term as a consequence of becoming richer. The education times of the IE and DE fall. The labor input of the UE rises initially but falls in the long term. The labor inputs of the IE and DE increase. The levels of human capital in the UD and IE are increased,

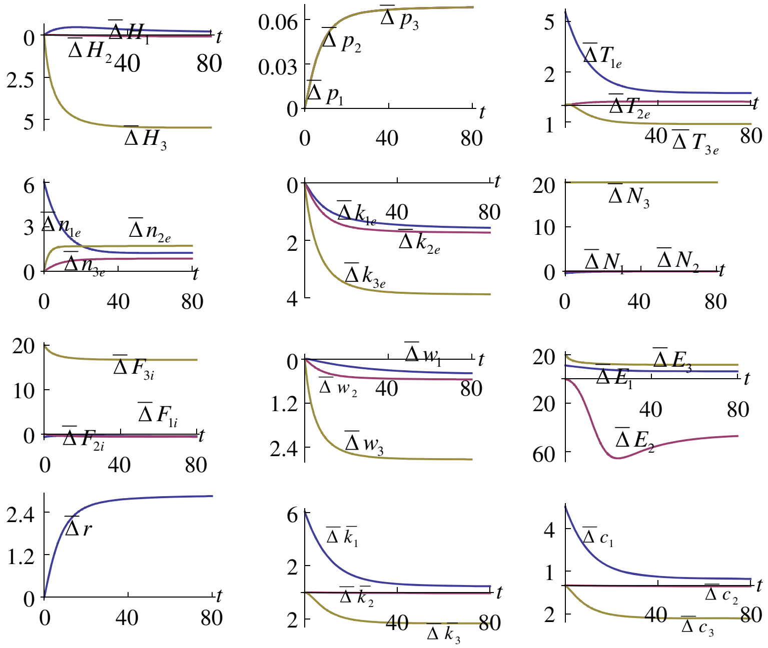

Fig. 4 A rise in the UE's population

but that of the DE is reduced. The wealth per capita of the UE rises over time, but the consumption level falls initially and rises in the long term. The UE's trade balance is improved, but the IE and the DE are deteriorated.

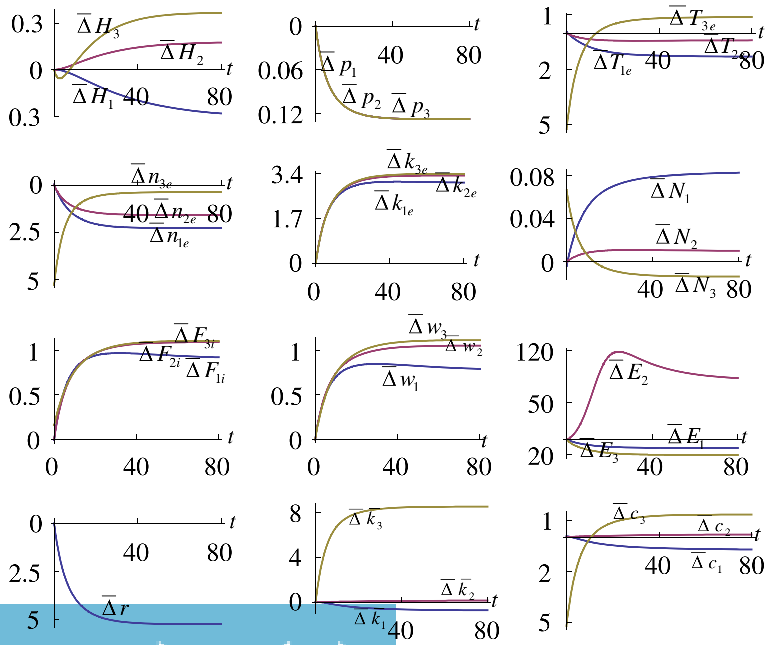

Fig. 5 A rise in the UE's propensity to save

Conclusions

This paper proposed a multicountry growth model with physical capital and human capital accumulation. Different from the growth models with the Ramsey approach in the literature, we used an alternative utility function to determine education, saving, and consumption. The dynamics of J-country world economy is controlled by $2J$ differential equations. An important contribution of the model is that it integrates the main ideas in different models in the literature of economic growth and international trade in a comprehensive framework. The model is for any number of countries, more general than almost all the trade models with growth which are only concerned with two countries. Our model is important as it deals with dynamic interdependence between endogenous physical and human capital accumulation for a fully globally connected world economy. In the literature of dynamic international trade with microeconomic foundation, the trade models study either endogenous capital accumulation or human capital accumulation, but few deal with both within an integrated framework. Because our model provides a general framework for global economic growth, it can provide some unique insights into economic globalization. For instance, as mentioned before, much of the discussion of income convergence is based on the insights from analyzing models of closed economies. One well-known prediction is that convergence in income levels among closed countries is achieved by faster accumulation of physical capital in the poor countries. This study does not predict this convergence in a globally connected economy. The poor may remain being poor even if they have the same propensity to save and to consume with the rich. Differences in education attitudes, learning ability, and depreciation of human capital between the rich and the poor may diverge economies over time (as the modern economic history shows). In a model without human capital depreciation, Tamura (1991, pp. 522–523) concludes: “Income convergence arises from human capital convergence… Convergence arises because below-average human capital agents gain the most from learning.” Our model predicts that human capital may not converge in the long term because there is depreciation of human capital. Hence, if a country has no ability to learn fast, it can never catch up.

We also simulated the motion of the model with three countries and carried out comparative dynamic analysis with regard to some parameters. It is well-known that the one-sector neoclassical growth model has been generalized and extended in many directions. It is not difficult to generalize our model along these lines. We may analyze the behavior of the model with other forms of production or utility functions. Another important extension of the paper is to introduce educational policies and interregional gaps in education and income within each country. In regional sciences, there are many empirical studies on education, but there are only a few theoretical (formal modeling) studies. For instance, Quadrado et al. (2001) study the effects of changes in educational policies upon interregional and intraregional inequality with respect to educational facilities in Spain. Hendricks (2010) examines how the variation in education across countries and within countries is primarily due to industry composition or to within-industry skill intensities. The study reveals that within-industry variation accounts for at least two thirds of the cross-country and the time series variation in educational attainment. It suggests that theories of educational development should focus on skills upgrading within industries rather than structural change. Winters (2011) examines the effects of the local human capital and the presence of higher education institutions on the quality of life in US metropolitan areas. Caniëls and Bosch (2011) provide a comprehensive review on the literature on the regional impact of the higher education institutes on regional innovation systems. Other topics related to regional economics and education can be found in the articles just cited.

Acknowledgments The author wishes to thank the editor Elias G. Carayannis and two anonymous referees for the valuable comments and constructive suggestions.

Appendix: Proving Lemma 1

First, from Eq. 2, we obtain:

$$k_{ji} = \phi_{ji}\left(k_{1i}, H_1, H_j\right) \equiv \left(\frac{\alpha_{ji} A_{ji} H_j^{m_j \beta_{ji}}}{\alpha_{1i} A_{1i} H_1^{m_1 \beta_{1i}} k_{1i}^{-\beta_{1i}} + \delta_j}\right)^{1/\beta_{ji}}, \quad j = 1, \ldots, J \tag{18}$$

where $\delta_j \equiv \delta_{k1} - \delta_{kj}$. It should be noted that $\phi_{1i} = k_{1i}$. From Eqs. 2 and 18, we determine the rate of interest and the wage rates as functions of $k_{1i}(t)$ and $H_j(t)$ as follows:

$$r = \overline{\phi}(k_{1i}, H_1) \equiv \alpha_{1i} A_{1i} H_1^{m_1 \beta_{1i}} k_{1i}^{-\beta_{1i}} - \delta_{1k}, \; w_j = \overline{\phi}_j\left(k_{1i}, H_j\right) \equiv \beta_{ji} A_{ji} H_j^{m_j \beta_{ji}} k_{ji}^{\alpha_{ji}}, \quad j = 1, \ldots, J. \tag{19}$$

From Eqs. 2 and 4, we obtain:

$$\frac{K_{je}}{N_{je}} = \alpha \frac{K_{ji}}{N_{ji}}, \text{ i.e., } \mathrm{k}_{je} = \alpha_j \mathrm{k}_{ji} \tag{20}$$

where $\alpha_j \equiv \alpha_{je}\beta_{ji} / \alpha_{ji}\beta_{je}$ ($\neq 1$ assumed). We also determine k_{je} as functions of k_{1i} and H_j. From Eqs. 20, 2, and 4, we obtain:

$$p_j = \frac{\alpha_{ji} A_{ji}}{\alpha_{je} A_{je}} \alpha_j^{\beta_j} H_j^{-m_j \beta_j} k_{ji}^{\beta_j} \tag{21}$$

where $\beta_j = \beta_{je} - \beta_{ji}$.

From Eqs. 20 and 1, we solve the labor distribution as functions of k_{ji} and k_j:

$$n_{ji} = \frac{\alpha_j k_{ji} - k_j}{\left(\alpha_j - 1\right) k_{ji}}, \; n_{je} = \frac{k_j - k_{ji}}{\left(\alpha_j - 1\right) k_{ji}}. \tag{22}$$

From Eqs. 12 and 4, we have:

$$T_{je} = A_{je} n_{je} T_j k_{je}^{\alpha_{je}} H_j^{\beta_{je} m_j} \tag{23}$$

where we also use the definitions of k_{je} and n_{je}. Insert $T_j + T_{je} = T_0$ and n_e in Eq. 22 into the preceding equation:

$$T_j = T_0 \left[1 + \frac{\tilde{\alpha}_j H_j^{\beta_{je} m_j}\left(k_j - k_{ji}\right)}{k_{je}^{\beta_{je}}}\right]^{-1} \tag{24}$$

where $\tilde{\alpha}_j \equiv A_{je}/(\alpha_j - 1)$. From $(p_j + w_j)T_{je} = \eta_j \bar{y}_j$ in Eq. 9 and the definition of $\bar{y}_j$, we have:

$$T_{je} = \varphi_{jp}\bar{k}_j + \varphi_{j0}, \tag{25}$$

where:

$$\varphi_{jp}(k_{1i}, H_1, H_j) \equiv \frac{(1+r)\eta_j}{p_j + w_j}, \quad \varphi_{j0}(k_{1i}, H_1, H_j) \equiv \frac{\eta_j T_0 w_j}{p_j + w_j}.$$

Insert T_j+T_{je}=T_0 and Eq. 24 in the preceding equation:

$$k_j = \varphi_{jk}(k_{1i}, H_1, H_j, \bar{k}_j) \equiv \left(\frac{\varphi_{jp}\bar{k}_j + \varphi_{j0}}{T_0 - \varphi_{jp}\bar{k}_j - \varphi_{j0}}\right)\frac{k_{je}^{\beta_{je}}}{\tilde{\alpha}_j H_j^{\beta_{je}m_j}} + k_{ji}. \tag{26}$$

From $T_j + T_{je} = T_0$ and Eq. 25, we have:

$$T_j = T_0 - \varphi_{jp}\bar{k}_j - \varphi_{j0}. \tag{27}$$

From $K_j = k_j T_j \bar{N}_j$ and Eqs. 26 and 27, we have:

$$K_j = (\varphi_{jp}\bar{k}_j + \varphi_{j0})\frac{\bar{N}_j k_{je}^{\beta_{je}}}{\tilde{\alpha}_j H_j^{\beta_{ej}m_j}} + (T_0 - \varphi_{jp}\bar{k}_j - \varphi_{j0})\bar{N}_j k_{ji}. \tag{28}$$

Insert Eq. 28 in Eq. 13:

$$\bar{k}_1 = \Lambda_k(k_{1i}, (H_j), \{\bar{k}_j\}) \equiv \left[\Lambda_0 + \frac{\varphi_{10}k_{1e}^{\beta_{1e}}}{\tilde{\alpha}_1 H_1^{\beta_{1e}m_1}} + (T_0 - \varphi_{10})k_{1i}\right]\left(1 - \frac{\varphi_{1p}k_{1e}^{\beta_{1e}}}{\tilde{\alpha}_1 H_1^{\beta_{1e}m_1}} + \varphi_{1p}k_{1i}\right)^{-1} \tag{29}$$

in which:

$$\Lambda_0(k_{1i}, (H_j), \{\bar{k}_j\}) = \frac{1}{\bar{N}_1}\sum_{j=2}^{J}\bar{N}_j\left[\frac{(\varphi_{jp}\bar{k}_j + \varphi_{j0})k_{je}^{\beta_{je}}}{\tilde{\alpha}_j H_j^{\beta_{je}m_j}} + (T_0 - \varphi_{jp}\bar{k}_j - \varphi_{j0})k_{ji} - \bar{k}_j\right]$$

where $(H_j) \equiv (H_1, \cdots, H_J)$ and $\{\bar{k}_j\} \equiv (\bar{k}_2, \cdots, \bar{k}_J)$. It is straightforward to confirm that all the variables can be expressed as functions of k_{1i}, (H_j), and $\{\bar{k}_j\}$ by the following procedure: k_{ji} by Eq. 18→k_{je} by Eq. 20→r and w_j by Eq. 19→p by Eq. 21→ $\bar{k}_1$ by Eq. 29→K_j by Eq. 29→k_j by Eq. 26→T_j by Eq. 27→T_{je} by Eq. 25→$N_j = T_j\bar{N}_j$ →n_{ji} and n_{je} by Eq. 22→N_{ji}=$n_{ji}N_j$ and N_{je}=$n_{je}N_j$→K_{ji}=$k_{ji}N_{ji}$ and K_{je}=$k_{je}N_{je}$→F_{ji} by Eq. 2→F_{je} by Eq. 12→$\bar{y}_j$ by Eq. 7→c_j and s_j by Eq. 9. From this procedure and Eq. 11, we have:

$$\dot{H}_j = \Lambda_j\left(k_{1i}, (H_j), \{\overline{k}_j\}\right) \equiv \frac{\upsilon_{je} F_{je}^{a_{je}} \left(H_j^{m_j} T_{je} \overline{N}_j\right)^{b_{je}}}{H_j^{\pi_{je}} \overline{N}_j} + \frac{\upsilon_{ji} F_{ji}^{a_{ji}}}{H_j^{\pi_{ji}} \overline{N}_j} + \frac{\upsilon_{jh} C_j^{a_{jh}}}{H_j^{\pi_h} \overline{N}_j} - \delta_{jh} H_j. \quad (30)$$

Here, we do not provide explicit expressions of the functions as they are tedious. Substituting $\overline{y}_j = (1+r)\overline{k}_j + T_0 w_j$ into $s_j = \lambda_j \overline{y}_j$ yields:

$$s_j = (1+r)\lambda_j \overline{k}_j + \lambda_j T_0 w_j. \quad (31)$$

Substituting Eq. 31 into Eq. 7, we have:

$$\dot{\overline{k}}_1 = \lambda_1 T_0 w_1 - R(k_{1i}, H_1)\overline{k}_1 \quad (32)$$

$$\dot{\overline{k}}_j = \overline{\Lambda}_j\left(k_{1i}, (H_j), \{\overline{k}_j\}\right) \equiv \lambda_j T_0 w_j - \left(1 - \lambda_j - \lambda_j r\right)\overline{k}_j, \quad j = 2, \ldots, J \quad (33)$$

in which $R(k_{1i}, H_1) \equiv 1 - \lambda_1 - \lambda_1 r$. Taking derivatives of Eq. 29 with respect to t yields:

$$\dot{\overline{k}}_1 = \frac{\partial \Lambda_k}{\partial k_{1i}} \dot{k}_{1i} + \sum_{j=1}^{J} \Lambda_j \frac{\partial \Lambda_k}{\partial H_j} + \sum_{j=2}^{J} \overline{\Lambda}_j \frac{\partial \Lambda_k}{\partial \overline{k}_j} \quad (34)$$

where we use Eqs. 30 and 33. Equaling the right-hand sizes of Eqs. 32 and 34, we get:

$$\dot{k}_{1i} = \overline{\Lambda}_1\left(k_{1i}, (H_j), \{\overline{k}_j\}\right) \equiv \left[\lambda_1 T_0 w_1 - R\Lambda_k - \sum_{j=1}^{J} \Lambda_j \frac{\partial \Lambda_k}{\partial H_j} - \sum_{j=2}^{J} \overline{\Lambda}_j \frac{\partial \Lambda_k}{\partial \overline{k}_j}\right] \left(\frac{\partial \Lambda_k}{\partial k_{1i}}\right)^{-1}. \quad (35)$$

In summary, we proved Lemma 1.

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